CHAPTER 14 Inverse Trigonometric Functions

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}.$$

Then $\tan\theta =$ ________ *(1981 - 2 Marks)*

2. The numerical value of $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to ________ *(1984 - 2 Marks)*

3. The greater of the two angles $A = 2\tan^{-1}(2\sqrt{2}-1)$ and $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$ is ________. *(1989 - 2 Marks)*

C MCQs with One Correct Answer

1. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is *(1983 - 1 Mark)*

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{16}{7}$ (d) none

2. If we consider only the principle values of the inverse trigonometric functions then the value of

$\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$ is *(1994)*

(a) $\frac{\sqrt{29}}{3}$ (b) $\frac{29}{3}$ (c) $\frac{\sqrt{3}}{29}$ (d) $\frac{3}{29}$

3. The number of real solutions of

$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$ is *(1999 - 2 Marks)*

(a) zero (b) one (c) two (d) infinite

4. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$

for $0 < |x| < \sqrt{2}$, then x equals *(2001S)*

(a) 1/2 (b) 1 (c) –1/2 (d) –1

5. The value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$ is *(2004S)*

(a) 1/2 (b) 1 (c) 0 (d) –1/2

6. If $0 < x < 1$, then

$\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2} =$ *(2008)*

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) x

(c) $x\sqrt{1+x^2}$ (d) $\sqrt{1+x^2}$

7. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is *(JEE Adv. 2013)*

(a) $\frac{23}{25}$ (b) $\frac{25}{23}$ (c) $\frac{23}{24}$ (d) $\frac{24}{23}$

D MCQs with One or More than One Correct

1. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is *(1986 - 2 Marks)*

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) none

2. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are) *(JEE Adv. 2015)*

(a) $\cos\beta > 0$ (b) $\sin\beta < 0$

(c) $\cos(\alpha+\beta) > 0$ (d) $\cos\alpha < 0$

E Subjective Problems

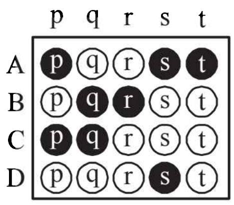
1. Find the value of : $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$, where $0 \le \cos^{-1} x \le \pi$ and $-\pi/2 \le \sin^{-1} x \le \pi/2$. *(1981 - 2 Marks)*

2. Find all the solution of $4\cos^2 x \sin x - 2\sin^2 x = 3\sin x$ *(1983 - 2 Marks)*

3. Prove that $\cos \tan^{-1} \sin \cot^{-1} x = \sqrt{\frac{x^2+1}{x^2+2}}$. *(2002 - 5 Marks)*

F Match the Following

DIRECTIONS (Q. 1 & 2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*



If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

1. Match the following *(2006 - 6M)*

Column I	*Column II*
(A) $\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then tan t =	(p) 1
(B) Sides a, b, c of a triangle ABC are in AP and $\cos\theta_1 = \frac{a}{b+c}, \cos\theta_2 = \frac{b}{a+c}, \cos\theta_3 = \frac{c}{a+b}$, then $\tan^2\left(\frac{\theta_1}{2}\right) + \tan^2\left(\frac{\theta_3}{2}\right) =$	(q) $\frac{\sqrt{5}}{3}$
(C) A line is perpendicular to $x + 2y + 2z = 0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(r) $\frac{2}{3}$

2. Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$. *(2007)*

Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	*Column II*
(A) If $a = 1$ and $b = 0$, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(B) If $a = 1$ and $b = 1$, they (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If $a = 1$ and $b = 2$, then (x, y)	(r) lies on $y = x$
(D) If $a = 2$ and $b = 2$, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

DIRECTIONS (Q. 3) : *Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

3. Match List I with List II and select the correct answer using the code given below the lists : ***(JEE Adv. 2013)***

List I	**List II**
P. $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right)^{1/2}$ takes value	1. $\frac{1}{2}\sqrt{\frac{5}{3}}$
Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\frac{x-y}{2}$ is	2. $\sqrt{2}$
R. If $\cos\left(\frac{\pi}{4}-x\right)\cos 2x + \sin x \sin 2 \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4}+x\right)\cos 2x$ then possible value of $\sec x$ is	3. $\frac{1}{2}$
S. If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x \neq 0,$ then possible value of x is	4. 1

Codes:

	P	**Q**	**R**	**S**
(a)	4	3	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	3	4	1	2

Section-B JEE Main / AIEEE

1. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$ **[2002]**

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

2. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for **[2003]**

(a) $|a| \ge \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

(c) all real values of a (d) $|a| < \frac{1}{2}$

3. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to **[2005]**

(a) $2\sin 2\alpha$ (b) 4

(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

4. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the values of x is

(a) 4 (b) 5 **[2007]**

(c) 1 (d) 3

5. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$

(c) $\frac{4}{17}$ (d) $\frac{5}{17}$

6. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then **[JEE M 2013]**

(a) $x = y = z$ (b) $2x = 3y = 6z$

(c) $6x = 3y = 2z$ (d) $6x = 4y = 3z$

7. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is : **[JEE M 2015]**

(a) $\frac{3x - x^3}{1 + 3x^2}$ (b) $\frac{3x + x^3}{1 + 3x^2}$

(c) $\frac{3x - x^3}{1 - 3x^2}$ (d) $\frac{3x + x^3}{1 - 3x^2}$

14 Inverse Trigonometric Functions

Section-A : JEE Advanced/ IIT-JEE

A 1. 0 2. $\frac{-7}{17}$ 3. A

C 1. (d) 2. (d) 3. (c) 4. (b) 5. (d) 6. (c) 7. (b)

D 1. (d) 2. (b, c, d)

E 1. $\frac{-2\sqrt{6}}{5}$ 2. $x = n\pi,\ n\pi + (-1)^n \frac{\pi}{10}, n\pi + (-1)^n\left(\frac{-3\pi}{10}\right)$ where $n \in N$

F 1. (A)-(p), (B)-(r), (C)-(q) 2. (A)-p, (B)-q, (C)-p, (D)-s 3. (b)

Section-B : JEE Main/ AIEEE

1. (a) 2. None 3. (c) 4. (d) 5. (a) 6. (a) 7. (c)

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Let $a+b+c=u$, then

$$\theta = \tan^{-1}\sqrt{\frac{au}{bc}} + \tan^{-1}\sqrt{\frac{bu}{ca}} + \tan^{-1}\sqrt{\frac{cu}{ab}}$$

Now we know that

$$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ when } xy > 1$$

$$\sqrt{\frac{au}{bc}} \times \sqrt{\frac{bu}{ca}} = \frac{u}{c} = \frac{a+b+c}{c} > 1; a, b, c$$

being +ve real no's.

$\therefore$ We get $$\theta = \pi + \tan^{-1}\left[\frac{\sqrt{\frac{au}{bc}} + \sqrt{\frac{bu}{ca}}}{1 - \sqrt{\frac{au}{bc}}\sqrt{\frac{bu}{ca}}}\right] + \tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$\theta = \pi + \tan^{-1}\left[\frac{\frac{a+b}{\sqrt{abc}}\sqrt{u}}{1-\frac{u}{c}}\right] + \tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$\theta = \pi + \tan^{-1}\left[\frac{(u-c)\sqrt{u}}{\sqrt{abc}} \times \frac{c}{-(u-c)}\right] + \tan^{-1}\sqrt{\frac{cu}{ab}}$$

$$\theta = \pi - \tan^{-1}\sqrt{\frac{uc}{ab}} + \tan^{-1}\sqrt{\frac{cu}{ab}}$$

[Using $\tan^{-1}(-x) = -\tan^{-1}x$] $= \pi$

$\therefore \tan\theta = \tan\pi = 0$

2. $$\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = \tan\left[\tan^{-1}\left(\frac{2/5}{1-(1/5)^2}\right) - \tan^{-1}(1)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{5}{12}\right) - \tan^{-1}(1)\right] = \tan\left[\tan^{-1}\left(\frac{5/12-1}{1+5/12}\right)\right]$$

$$= \tan(\tan^{-1}(-7/17)) = -7/17$$

3. We have

$$A = 2\tan^{-1}(2\sqrt{2}-1) = 2\tan^{-1}(2 \times 1.414 - 1)$$

$$= 2\tan^{-1}(1.828) > 2\tan^{-1}\sqrt{3} = 2\pi/3$$

$\Rightarrow A > 2\pi/3$ ...(1)

Also $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$

$$= \sin^{-1}\left[3 \times \frac{1}{3} - 4 \times \frac{1}{27}\right] + \sin^{-1}(3/5)$$

$$= \sin^{-1}\frac{23}{27} + \sin^{-1}(0.6) = \sin^{-1}(0.852) + \sin^{-1}(0.6)$$

$$< \sin^{-1}(\sqrt{3}/2) + \sin^{-1}(\sqrt{3}/2) = 2.\pi/3$$

$\Rightarrow B < 2\pi/3$ ....(2)

From (1) and (2) we conclude $A > B$.

C. MCQs with ONE Correct Answer

1. (d) $\tan\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$

$= \tan\left[\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right]$

$= \tan\left[\tan^{-1}\left(\frac{3/4+2/3}{1-3/4\times 2/3}\right)\right]=\frac{17}{12}\times\frac{12}{6}=\frac{17}{6}$

2. (d) $\tan\left[\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right]$

$= \tan[\tan^{-1}7-\tan^{-1}4]$

$= \tan\left(\tan^{-1}\left(\frac{3}{29}\right)\right)=\frac{3}{29}$

3. (c) $\tan^{-1}\sqrt{[x(x+1)]}=\pi/2-\sin^{-1}\sqrt{(x^2+x+1)}$

$\Rightarrow \tan^{-1}\sqrt{x(x+1)}=\cos^{-1}\sqrt{x^2+x+1}$

$\Rightarrow \cos^{-1}\frac{1}{\sqrt{x^2+x+1}}=\cos^{-1}\sqrt{x^2+x+1}$

$\Rightarrow x^2+x+1=1 \Rightarrow x(x+1)=0$

$\Rightarrow x=0,-1$ are the only real solutions.

4. (b) $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{3}...\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}...\right)=\frac{\pi}{2}$

$\Rightarrow \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}...\right)=\frac{\pi}{2}-\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}...\right)$

$\Rightarrow \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}...\right)=\cos^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}...\right)$

$\Rightarrow x^2-\frac{x^4}{2}+\frac{x^6}{4}...=x-\frac{x^2}{2}+\frac{x^3}{4}...$

On both sides we have G.P. of infinite terms.

$\therefore \frac{x^2}{1-\left(\frac{-x^2}{2}\right)}=\frac{x}{1-\left(-\frac{x}{2}\right)}\Rightarrow\frac{2x^2}{2+x^2}=\frac{2x}{2+x}$

$\Rightarrow 2x+x^3=2x^2+x^3\Rightarrow x(x-1)=0$

$\Rightarrow x=0,1$ but $0<|x|<\sqrt{2}\Rightarrow x=1.$

5. (d) $\sin\left[\cot^{-1}(1+x)\right]=\cos(\tan^{-1}x)$

$\Rightarrow \sin\left[\sin^{-1}\left(\frac{1}{\sqrt{1+(1+x)^2}}\right)\right]=\cos\left[\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$

$\Rightarrow \frac{1}{\sqrt{1+(1+x)^2}}=\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow 1+1+2x+x^2=1+x^2\Rightarrow 2x+1=0\Rightarrow x=-\frac{1}{2}$

6. (c) $\sqrt{1+x^2}\left[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1\right]^{\frac{1}{2}}$

$=\sqrt{1+x^2}\left[\left\{x\cos\left(\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)+\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)\right\}^2-1\right]^{\frac{1}{2}}$

$=\sqrt{1+x^2}\left[\left\{x.\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]$

$=\sqrt{1+x^2}\left[\left(\sqrt{1+x^2}\right)^2-1\right]=x\sqrt{1+x^2}$

7. (b) $\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)=\cot^{-1}[1+n(n+1)]$

$=\tan^{-1}\left[\frac{(n+1)-n}{1+(n+1)n}\right]=\tan^{-1}(n+1)-\tan^{-1}n$

$\therefore \sum_{n=1}^{23}[\tan^{-1}(n+1)-\tan^{-1}n]=\tan^{-1}24-\tan^{-1}1=\tan^{-1}\frac{23}{25}$

$\therefore \cot\left[\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right]=\cot\left[\tan^{-1}\frac{23}{25}\right]=\frac{25}{23}$

D. MCQs with ONE or MORE THAN ONE Correct

1. (d) The principal value of

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$

$=\sin^{-1}(\sin\pi/3)=\pi/3 \therefore$ (d) is the correct answer.

2. (b, c, d)

$\alpha=3\sin^{-1}\frac{6}{11}>3\sin^{-1}\frac{1}{2}$ or $\alpha>\frac{\pi}{2}$

$\therefore \cos\alpha<0$

$\beta=3\cos^{-1}\frac{4}{9}>3\cos^{-1}\frac{1}{2}$ or $\beta>\pi$

$\therefore \cos\beta<0$ and $\sin\beta<0$

Also $\alpha+\beta>\frac{3\pi}{2}\ \therefore \cos(\alpha+\beta)>0.$

E. SUBJECTIVE PROBLEMS

1. We have $\cos(2\cos^{-1}x+\sin^{-1}x)$

$= \cos(\cos^{-1}x+\cos^{-1}x+\sin^{-1}x)$

$= \cos(\cos^{-1}x+\pi/2)$ {Using $\cos^{-1}x+\sin^{-1}x=\pi/2$}

$= -\sin(\cos^{-1}x)$

$= -\sqrt{1-\cos^2(\cos^{-1}x)} = -\sqrt{1-[\cos(\cos^{-1}x)]^2}$

$= -\sqrt{1-x^2} = -\sqrt{1-1/25}$ [for $x = 1/5$]

$= -\dfrac{\sqrt{24}}{5} = \dfrac{-2\sqrt{6}}{5}$

2. Given eq. is,

$4\cos^2x\sin x-2\sin^2x=3\sin x$

$\Rightarrow 4\cos^2x\sin x-2\sin^2x-3\sin x=0$

$\Rightarrow 4(1-\sin^2x)\sin x-2\sin^2x-3\sin x=0$

$\Rightarrow \sin x[4\sin^2x+2\sin x-1]=0$

$\Rightarrow$ either $\sin x=0$ or $4\sin^2x+2\sin x-1=0$

If $\sin x=0 \Rightarrow x=n\pi$

$\Rightarrow$ If $4\sin^2x+2\sin x-1=0 \Rightarrow \sin x=\dfrac{-1\pm\sqrt{5}}{4}$

If $\sin x=\dfrac{-1\pm\sqrt{5}}{4}=\sin 18°=\sin\dfrac{\pi}{10}$

then $x = nx+(-1)^4\dfrac{\pi}{10}$

If $\sin x = -\left(\dfrac{\sqrt{5}+1}{4}\right)=\sin(-54°)=\sin\left(\dfrac{-3\pi}{10}\right)$

then $x = n\pi+(-1)^n\left(\dfrac{-3\pi}{10}\right)$

Hence, $x = n\pi,\ n\pi+(-1)^n\dfrac{\pi}{10}$ or $n\pi+(-1)^n\left(\dfrac{-3\pi}{10}\right)$

where n is some integer

3. To prove that $\cos\tan^{-1}\sin\cot^{-1}x=\sqrt{\dfrac{x^2+1}{x^2+2}}$.

L.H.S. $= \cos[\tan^{-1}(\sin(\cot^{-1}x))]$

$= \cos[\tan^{-1}(\sin(\sin^{-1}\dfrac{1}{\sqrt{1+x^2}}))]$ if $x>0$

and $\cos[\tan^{-1}(\sin(\pi-\sin^{-1}\dfrac{1}{\sqrt{1+x^2}}))]$ if $x<0$

In each case,

$= \cos\left[\tan^{-1}\dfrac{1}{\sqrt{1+x^2}}\right]=\cos\left[\cos^{-1}\sqrt{\dfrac{1+x^2}{2+x^2}}\right]$

$= \sqrt{\dfrac{x^2+1}{x^2+2}}=R.H.S.$ *Hence Proved.*

F. Match the Following

1. **(A) → (p); (B) → (r); (C) → (q)**

(A) $t=\sum_{i=1}^{\infty}\tan^{-1}\left(\dfrac{1}{2i^2}\right)=\sum_{i=1}^{\infty}\tan^{-1}\left[\dfrac{(2i+1)-(2i-1)}{1+4i^2-1}\right]$

$= \sum_{i=1}^{\infty}[\tan^{-1}(2i+1)-\tan^{-1}(2i-1)]$

$t=\tan^{-1}3-\tan^{-1}1+\tan^{-1}5-\tan^{-1}3$
$+ ......+\tan^{-1}(2n+1)-\tan^{-1}(2n-1)+......\infty$

$\Rightarrow t=\lim_{n\to\infty}[\tan^{-1}(2n+1)-\tan^{-1}1]$

$= \lim_{n\to\infty}\tan^{-1}\left[\dfrac{2n}{1+(2n+1)}\right]=\lim_{n\to\infty}\tan^{-1}\left[\dfrac{1}{1+1/n}\right]$

$\Rightarrow t=\tan^{-1}(1)=\dfrac{\pi}{4}\Rightarrow \tan t=1$, (A) → (p)

(B) $\because a, b, c$ are in AP $\Rightarrow 2b=a+c$

$\cos\theta_1=\dfrac{a}{b+c}$

$\Rightarrow \dfrac{1-\tan^2\theta_1/2}{1+\tan^2\theta_1/2}=\dfrac{a}{b+c}\Rightarrow \tan^2\dfrac{\theta_1}{2}=\dfrac{b+c-a}{b+c+a}$

Similarly, $\tan^2\dfrac{\theta_3}{2}=\dfrac{a+b-c}{a+b+c}$

$\Rightarrow \tan^2\dfrac{\theta_1}{2}+\tan^2\dfrac{\theta_3}{2}=\dfrac{2b}{a+b+c}=\dfrac{2b}{3b}=\dfrac{2}{3}$, (B) → (r)

(C) Equation of line through (0, 1, 0) and perpendicular to

$x+2y+2z=0$ is $\dfrac{x}{1}=\dfrac{y-1}{2}=\dfrac{z}{2}=\lambda$

For some value of λ, the foot of perpendicular from origin to line is $(\lambda, 2\lambda+1, 2\lambda)$

Dr's of this ⊥ from origin are $\lambda, 2\lambda+1, 2\lambda$

$\therefore 1.\lambda+2(2\lambda+1)+2.2\lambda=0\Rightarrow\lambda=-\dfrac{2}{9}$

$\therefore$ Foot of perpendicular is $\left(-\dfrac{2}{9},\dfrac{5}{9},-\dfrac{4}{9}\right)$

$\therefore$ Required distance

$= \sqrt{\dfrac{4}{81}+\dfrac{25}{81}+\dfrac{16}{81}}=\sqrt{\dfrac{45}{81}}=\dfrac{\sqrt{5}}{3}$ (C) → (q)

2. **(A) → p, (B) → q, (C) → p, (D) → s**

$\sin^{-1}(ax)+\cos^{-1}y+\cos^{-1}(bxy)=\dfrac{\pi}{2}$

$\Rightarrow \cos^{-1}y+\cos^{-1}(bxy)=\dfrac{\pi}{2}-\sin^{-1}(ax)=\cos^{-1}(ax)$

Let $\cos^{-1}y=\alpha$, $\cos^{-1}(bxy)=\beta$, $\cos^{-1}(ax)=\gamma$

$\Rightarrow$ $y = \cos\alpha$, $bxy = \cos\beta$, $ax = \cos\gamma$

$\therefore$ We get $\alpha + \beta = \gamma$ and $\cos\beta = bxy$

$\Rightarrow$ $\cos(\gamma - \alpha) = bxy$

$\Rightarrow$ $\cos\gamma\cos\alpha + \sin\gamma\sin\alpha = bxy$

$\Rightarrow$ $axy + \sin\gamma\sin\alpha = bxy \Rightarrow (a-b)xy = -\sin\alpha\sin\gamma$

$\Rightarrow$ $(a-b)^2x^2y^2 = -\sin^2\alpha\sin^2\gamma$

$= (1-\cos^2\alpha)(1-\cos^2\gamma)$

$\Rightarrow$ $(a-b)^2x^2y^2 = (1-a^2x^2)(1-y^2)$ ....(1)

(A) For $a = 1, b = 0$, equation (1) reduces to
$x^2y^2 = (1-x^2)(1-y^2) \Rightarrow x^2 + y^2 = 1$

(B) For $a = 1$, $b = 1$ equation (1) becomes
$(1-x^2)(1-y^2) = 0 \Rightarrow (x^2-1)(y^2-1) = 0$

(C) For $a = 1, b = 2$ equation (1) reduces to
$x^2y^2 = (1-x^2)(1-y^2) \Rightarrow x^2 + y^2 = 1$

(D) For $a = 2$, $b = 2$ equation (1) reduces to
$0 = (1-4x^2)(1-y^2) \Rightarrow (4x^2-1)(y^2-1) = 0$

3. **(b)**

(P) $$\left[\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y) + y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y) + \tan(\sin^{-1}y)}\right)^2 + y^4\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{y^2}\left(\frac{\cos\left(\cos^{-1}\frac{1}{\sqrt{1+y^2}}\right) + y\sin\left(\sin^{-1}\frac{y}{\sqrt{1+y^2}}\right)}{\cot\left(\cot^{-1}\frac{\sqrt{1-y^2}}{y}\right) + \tan\left(\tan^{-1}\frac{y}{\sqrt{1-y^2}}\right)}\right)^2 + y^4\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}}{\frac{1}{y(\sqrt{1-y^2})}}\right)^2 + y^4\right]^{\frac{1}{2}}$$

$= (1 - y^4 + y^4)^{\frac{1}{2}} = 1$ $\therefore$ (P) $\to$ (4)

(Q) We have $\cos x + \cos y = -\cos z$
$\sin x + \sin y = -\sin z$
Squaring and adding we get

$(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = \cos^2 z + \sin^2 z$

$\Rightarrow$ $2 + 2\cos(x-y) = 1$

$\Rightarrow$ $4\cos^2\frac{x-y}{2} = 1$ or $\cos\frac{x-y}{2} = \frac{+1}{2}$

$\therefore$ Q $\to$ (3)

(R) We have

$$\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x$$

$$= \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$$

$$\Rightarrow \cos 2x\left[\cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{\pi}{4} + x\right)\right]$$

$$= \sin 2x \sec x(\cos x - \sin x)$$

$$\Rightarrow 2\sin\frac{\pi}{4}\sin x\cos 2x = 2\sin x(\cos x - \sin x)$$

$$\Rightarrow 2\sin x\left[\frac{1}{\sqrt{2}}(\cos^2 x - \sin^2 x) - (\cos x - \sin x)\right] = 0$$

$$\Rightarrow 2\sin x(\cos x - \sin x)\left(\frac{\cos x + \sin x}{\sqrt{2}} - 1\right) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \tan x = 1 \text{ or } \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\Rightarrow x = 0, \frac{x}{4} \Rightarrow \sec x = 1 \text{ or } \sqrt{2}$$

$\therefore$ (R) $\to$ (2)

(S) $$\cos\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}x\sqrt{6}\right)$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{1+6x^2}} \Rightarrow x = \pm\frac{5}{2\sqrt{3}}$$

$\therefore$ (S) $\to$ (1)

Hence (P) $\to$ (4), (Q) $\to$ (3), (R) $\to$ (2), (S) $\to$ (1)

Section-B JEE Main/ AIEEE

1. **(a)** $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$$\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$$

$$\Rightarrow \tan^{-1}\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan^{-1}\frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = x$$

$$\Rightarrow \tan x = \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} \text{ or } \cot x = \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha}$$

$$\Rightarrow \sin x = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{1-(1-2\sin^2\alpha/2)}{1+2\cos^2\alpha/2-1}$$

or $\sin x = \tan^2\frac{\alpha}{2}$

2. **(None)** $\sin^{-1} x = 2\sin^{-1} a$

$$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2};\ \therefore -\frac{\pi}{2} \le 2\sin^{-1} a \le \frac{\pi}{2}$$

$$-\frac{\pi}{4} \le \sin^{-1} a \le \frac{\pi}{4} \text{ or } \frac{-1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$$

$$\therefore |a| \le \frac{1}{\sqrt{2}}$$

Out of given four options none is absolutely correct

3. **(c)** $\cos^{-1} x - \cos^{-1}\frac{y}{2} = \alpha$

$$\cos^{-1}\left(\frac{xy}{2} + \sqrt{(1-x^2)\left(1-\frac{y^2}{4}\right)}\right) = \alpha$$

$$\cos^{-1}\left(\frac{xy + \sqrt{4-y^2-4x^2+x^2y^2}}{2}\right) = \alpha$$

$$\Rightarrow 4 - y^2 - 4x^2 + x^2y^2$$

$$= 4\cos^2\alpha + x^2y^2 - 4xy\cos\alpha$$

$$\Rightarrow 4x^2 + y^2 - 4xy\cos\alpha = 4\sin^2\alpha.$$

4. **(d)** $\sin^{-1}\left(\frac{x}{5}\right) + \mathrm{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \mathrm{cosec}^{-1}\left(\frac{5}{4}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$$

$[\because \sin^{-1} x + \cos^{-1} x = \pi/2]$

$$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5} \Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3$$

5. **(a)** $\cot\left(\mathrm{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right) = \cot\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right]$

$$= \cot\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right] = \cot\left[\tan^{-1}\frac{17}{6}\right]$$

$$= \cot\left(\cot^{-1}\frac{6}{17}\right) = \frac{6}{17}$$

6. **(a)** Since, x, y, z are in A.P. $\Rightarrow 2y = x + z$
Also, we have, $2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \quad (\because\ 2y = x+z)$$

$$\Rightarrow y^2 = xz \text{ or } x+z=0 \Rightarrow x=y=z$$

7. **(c)** $\tan^{-1}\mathrm{y} = \tan^{-1}\mathrm{x} + \tan^{-1}\left[\frac{2\mathrm{x}}{1-\mathrm{x}^2}\right]$

$$= \tan^{-1}\mathrm{x} + 2\tan^{-1}\mathrm{x} = 3\tan^{-1}\mathrm{x}$$

$$\tan^{-1}\mathrm{y} = \tan^{-1}\left[\frac{3\mathrm{x}-\mathrm{x}^3}{1-3\mathrm{x}^2}\right] \Rightarrow \mathrm{y} = \frac{3\mathrm{x}-\mathrm{x}^3}{1-3\mathrm{x}^2}$$